Fiverr order [redacted]:
Method Justification and Results Document

Fiverr Vendor ID: tzlange
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Request

- Client wants to determine how similar the thickness consistency of pots at each site are to the verified cooking pot.

Solution:

The consistency of a vessel's wall thickness would be reflected in its variance, where greater inconsistency would mean higher variance. The analytical approach taken is to compare the variance of vessels' horizontal wall thickness to that of a vessel with abundant evidence of cooking. I compare pots' variance in horizontal wall thickness to the verified cooking pot's (BauVes 116); pots with statistically equal variance can be reasonably categorized as cooking vessels.

Traditional F-tests of variance homogeneity assume that the variables follow a normal distribution with independent measurements. Deviation from these assumption can yield biased results and Type 1 statistical errors (false positives). To overcome this, Levene (1960) and Brown & Forsythe (1974) formulated equal variance test that are more robust to deviations from normality by variable transformation. Using the provided data as an example, consider a variable X_{iv} to be the wall thickness measurement *i* from vessel *v*. The transformation is:

$$Z_{iv} = |X_{iv} - \widetilde{X_v}|$$

where $\widetilde{X_v}$ is the measure of center (mean for Levene (1960) or median for Brown & Forsythe (1974)). The test statistic *W* compares the variance in thickness measures for 2 or more vessels

$$W = \frac{N-V}{V-1} * \frac{(\sum_{v=1}^{V} N_v(\overline{Z_{v.}} - \overline{Z_{..}})^2)}{\sum_{v=1}^{V}\sum_{i=1}^{N_v}(Z_{iv} - \overline{Z_{v.}})^2}$$

where V is the number of vessels, N is the total number of measurements taken for all vessels, N_v is the number of samples from vessel *v*, $\overline{Z_{v.}}$ Is the mean Z for group *v*, and $\overline{Z_{..}}$ is the global mean for Z. The resulting W statistic is F-distributed with V-1 and N-V degrees of freedom.

The variance in horizontal vessel wall thickness in the provided data also depends on which Birkhoff (1933, pp. 69–70) inflection point a measurement is drawn from. I account for this by altering the transformation to be conditional on Birkhoff point:

$$Z_i = |X_{ibv} - \widetilde{X_{bv}}|$$

where X_{ibg} is measurement *i* at Birkhoff point *b* for vessel *v*, and $\widetilde{X_{bv}}$ is the measure of center at Birkhoff point *b* for vessel *v*. I will refer to this transformation as the Birkhoff-conditional Z.

Brown-Forsythe tests, which use the median as the measure of center, have been found to have low type-1 error rates by both Brown & Forsythe (1974) and more recently Derrick et al., (2018), so I use the median to compute *Z*'s.

Results

The results of the modified brown-forsythe tests indicate that most vessels have statistically equal variance in horizontal vessel wall to the comparison, regardless of the site they were retrieved from. Table 1 shows a summary of wall thickness measurements of the BauVes 116 comparison vessel for the whole vessel and at each Birkhoff Point. BauVes 116 has an average Birkhoff-conditional *Z* of 0.36 and a variance of 0.178

Table 1 Summary Statistics of BauVes 116 Standard

Birkhoff Point	Median Measurement (cm)	Number of Measurements	Mean Z (cm)	Variance Z
1st End Point	5	5	0.4	0.3
Corner Point	11	5	0.4	0.3
1st Vertical Tangency	5.5	6	0.5	0.0
Inflection Point	6	6	0.222	0.194
Total	6	25	0.36	0.178

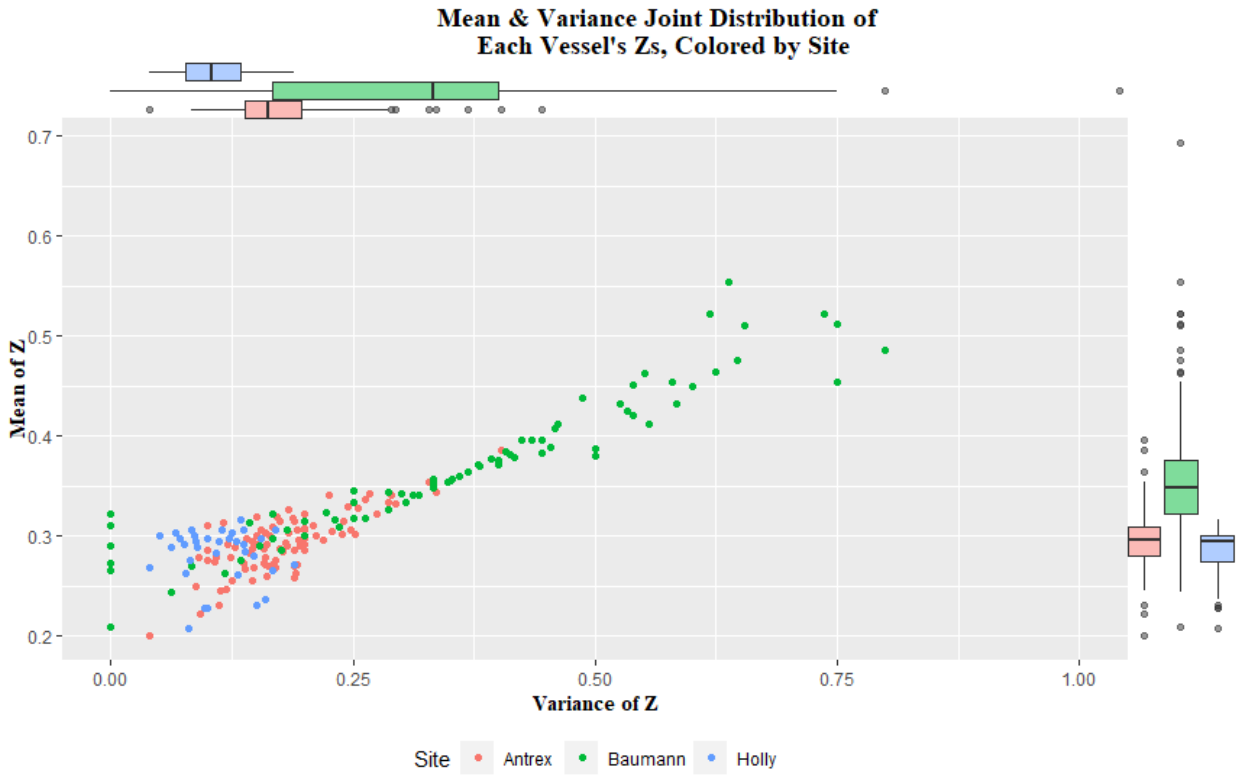

Figure 1 Joint distribution of the mean and variance of Birkhoff-conditional *Z* measurements for vessels found at Antrex, Baumann, and Holly. Box plots for each site's mean and variance border the main scatter plot to show each site's range more clearly for each site. The mean and variance for each site show a positive correlation. Holly and Antrex sites appear to have consistently small Variances and means, indicating that many of the vessels at each site have very consistent horizontal wall thickness. Vessels taken from the Baumann have more vessels with relatively high variance.

Figure 1 displays the joint distribution of averages and variances for vessels' Zs colored by site. Each point represents a vessel, and the scatter plot is flanked by box plots of each site's averages on the y axis and variances on the x axis. Z averages range from .2 to .69, and the variances range from 0 to 1.04. Vessels recovered from the Baumann site have a much larger range of both variances and means, while the Antrex and Holly sites have similar and relatively small averages and variances.

Table 2 Number of Vessels at each Site Less than, Greater than, or Equal to BauVess 116 at $\alpha = .05$

Site	N	Less Than	Equal To	Greater Than
Antrex	115	19 (16.5%)	93 (80.8%)	3 (2.7%)
Baumann	143	10 (6.9%)	124 (86.7%)	9 (6.4%)
Holly	40	10 (25.0%)	30 (75.0%)	0 (0.0%)
Total	298	39 (13.1%)	247 (82.0%)	12 (4.9%)

Table 2 displays site aggregated results of all modified Brown-Forsythe tests. 82% of vessels have statistically equivalent variances in birkoff-conditional Z horizontal wall thicknesses to the comparison vessel; 80.8% of vessels at antrex, 86.7% at Baumann, and 75% at Holly. 13.1% have variances that are statistically smaller (16.5% at Antrex, 6.9% at Baumann, and 25% at Holly). Finally, 4.9% have statistically greater variances than our comparison vessel; 2.7% at Antrex, 6.4% at Baumann, and 0% at Holly. See attached CSVs for vessel specific hypothesis tests.

References

Birkhoff, G. D. (1933). *Aesthetic Measure:* Harvard University Press.

Brown, M. B., & Forsythe, A. B. (1974). Robust Tests for the Equality of Variances. *Journal of the American Statistical Association*, *69*(346), 364–367. https://doi.org/10.2307/2285659

Derrick, B., Ruck, A., Toher, D., & White, P. (2018). Tests for equality of variances between two samples which contain both paired observations and independent observations. *Journal of Applied Quantitative Methods*, *13*(2). https://uwe-repository.worktribe.com/output/865549/tests-for-equality-of-variances-between-two-samples-which-contain-both-paired-observations-and-independent-observations

Levene, H. (1960). Robust tests for equality of variances. In I. Olkin & H. Hotelling (Eds.), *Contributions to Probability and Statistics: Essays in Honor of Harold Hotelling* (pp. 278–292). Stanford University Press.